Appendix C
Deriving the Equations in Chapter Eight

The derivations for several of the equations in Chapter Eight exceeded what I considered the average readers' mathematical knowledge. An explanation of these equations for the novice would have been too long and involved. (Concepts such as simultaneous solutions of equations, logarithms, and calculus must be clearly understood.) I therefore present the derivations of these equations below in the order in which they appeared within the book and assume the reader is familiar with the above concepts. The derivations presented in this appendix are those that I believe are the most straight forward. They are not intended to be the only solutions, indeed the reader may wish to find other insightful solutions.

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On page 156 the equation $y = 2.5t^2 + 5t$ is given for hypothetical earnings assuming a linear pay rate that increases \$5.00/h every hour. Two possible derivations for the above equation can be done as follows: solution of simultaneous equations, and the integration of $y = 5t + 5$. Both cases are shown below, respectively.

Case 1:

The general form of the quadratic equation is:
$f(x) = ax^2 + bx + c$

Table 8.1 gives f(0) = 0, f(1) = 7.5, and f(2) = 20. f(0) = 0 gives the value of the c as 0 (0 = a(0) + b(0) + c).

f(1) and f(2) are used to generate two equations in two unknowns:

f(1) = 7.5 = a(1) + b(1) = a + b

f(2) = 20 = a(2) + b(2) = 4a +2b, therefore our two equations are:

7.5 = a + b

20 = 4a + 2b

The solution for this set of simultaneous equations is (2.5,5), hence a = 2.5 and b = 5. Knowing a = 2.5, b = 5 and c = 0, we can write the quadratic equation $y = 2.5t^2 + 5t$.

Case 2:

The problem can also be solved by integrating the equation: y = 5t + 5. Since y represents the rate at which the earnings are acquired (the pay rate) we can express y as de/dt where e represents the earnings. The equation we need to integrate is therefore:

$\int de = \int (5t + 5)dt$

The right side of the equation integrates out to $2.5t^2 + 5t$. The initial condition: the earnings = 0 (e = 0) when t = 0, gives 0 for the constant of integration—which is the constant c in the quadratic equation. Once again our solution is $y = 2.5t^2 + 5t$, by a change of variable name.

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On page 159 the distance a freely falling object falls as a function of time is given as $d = 16t^2$. Using the same two methods described above we can quickly arrive at this equation. For the first method we use Table 8.2.

At t = 0 seconds the distance fallen is 0 feet, so:

0 = a(0) + b(0) + c, therefore c = 0 feet

Similarly, for t = 1 second, d = 16 feet, so:

16 = a(1) + b(1), and for t = 2 seconds, d = 64 feet, so 64 = a(2) + b(2) = 4a + 2b, therefore our two equations are:

16 = a + b

64 = 4a + 2b

Solving the two equations simultaneously gives b = 0 and a = 16. Hence, $d = 16t^2$. Similarly, evaluating $\int dx = \int 32t dt$ gives the same results (where the speed = $^{dx}/_{dt}$).

Let s = dx/dt, then: dx = 32t dt. The integral $\int 32t\, dt$ is $16t^2$. The constant of integration equals zero since x = 0 when t = 0. The final equation is therefore, $x = 16t^2$, or as before, by changing the variable name $d = 16t^2$.

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On page 166 we are given the equation $y = 7.21(2^X-1)$ as the earnings for the exponential pay rate $y = 5(2^X)$. The same methods can be used again to derive the first equation. However, integration of $y = 5(2^X)$ will provide for a more accurate answer since the 7.21 is an approximation. In general, the integral of an exponential function $\int a^X dx$ evaluates to: $a^X(1/\ln a) + C$, where ln is the natural log, a for our case is 2, and C is the constant of integration. 1 divided by the natural log of 2 (1/ln2) = 1.442695041.... This number is multiplied by the constant 5 which gives 7.213475205..., hence the constant 7.21 in the equation above. Our equation is therefore: $y = 7.21(2^X) + C$ (rounding-off 7.213475205...). Table 8.3 shows that when t = 0, y = 0. Plugging these values into the above equation gives, C = -7.21, which yields, $y = 7.21(2^X) - 7.21$, and upon factoring out the 7.21, leaves us with $y = 7.21(2^X- 1)$.

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Figure 8.6, page 168, shows Third World population as a non-linear function. If Third World population is plotted on semi-log paper the data will straighten out into an essentially linear looking relationship between time and population growth. This shows that

Third World population has and is expected to continue growing exponentially over the domain [1950,2000]. (If log paper is not available, the data can be linearized by taking the logarithm of dependent variable and plotting it against the independent variable.)