CHAPTER SIX

Three Problems — Diet, Inflation, and Taxes

The following problems are designed to help the reader become more familiar with the mathematics discussed in Chapter Five. Each problem provides for more than the one-dimensional view that most math problems are reduced to in high school and college textbooks. The problems are structured so that the mathematics is perceived as an "embedded fragment" (albeit an important fragment) rather than isolated from the information it is illuminating. The reader will find the questions posed to be topical and diverse. Hopefully, this chapter will help readers become more sensitive to the power, necessity, and versatility of mathematics.

Problem #1
Fat from Fiction

By mid-1994 Americans should be seeing a revolution in food labeling. The Nutrition Labeling and Education Act authorized the Food and Drug Administration (FDA) to create a meaningful and simple labeling system for all processed foods. At first glance, such news may seem minor compared to inflation and taxes. But what we eat is intimately tied to our economy, our environment, and our health. The decade of the nineties will set the stage for large scale shifts in dietary patterns that will redefine our preferences and our consumption.

Over the last five to ten years Americans have grown more concerned with the associated risks of heart disease and cancer due to the Standard American Diet (SAD), which has traditionally consisted of high-fat, low-fiber foods.

All of us were taught in school to eat a diet based on the four food groups. The first two groups are composed of meat and dairy products which are high in fat and low in fiber. If you're like most Americans, you remember those colorful charts used by your teachers to show the benefits of an animal food diet. All of these materials were provided free to our schools by those who had the most to gain by the consumption of these products. The National Dairy Council and the Beef Council are examples of industries that have spent millions of dollars to bring us the virtues of animal products. We never realized we were being handed a sales pitch instead of sound nutritional advice.

After World War II, animal product consumption rose dramatically in America.

There are several reasons for this:

1. Americans became more affluent and could afford eating higher on the food chain.
2. Grain production rose to high surplus levels, due to chemical innovations with fertilizers and pesticides. Therefore, it made sense economically to feed the surplus grain to animals, especially cows, because they ate the most and gave back the least.
3. The rise of agribusiness and the development of factory farming emerged. (Factory farming has little to do with farming in the traditional sense. It is a concentrated form of husbandry where tens of thousands of animals are kept in confined areas. The advent of strong antibiotics made intensive confinement possible. Without these drugs, such "farms" would be lost to epidemics of disease.)
4. The meat and dairy industries sold the public on the idea that animal products were indispensable to human health.

Studies over the past thirty years have indicated that a high-fat, low-fiber diet (which is usually a meat-centered diet) is not as healthy as those in the medical profession and food industry once proclaimed. One study conducted in the early 1970s showed a strong correlation between fat consumption and breast cancer.[1]

Accurate food labeling is the first line of defense for consumer health and for controlling the enormous cost of health care that every American shoulders through private funds and tax dollars. How much of a tax burden would be lifted from all of us if the incidence of expensive degenerative diseases such as cancer and heart disease were substantially reduced? John Robbins devoted a large portion of his book, *Diet for a New America* to the cause of preventative medicine. I have taken the liberty to quote a rather long passage from his book:

> Billions upon billions of dollars are being poured into the search for the 'magic bullet' that will cure cancer, a search that has thus far been utterly unsuccessful. And yet, at the same time, another search has been underway which has borne great fruit. Unbeknownst to the public, we have been learning more and more about how to prevent the disease in the first place.
>
> The tragedy is that the American people have been continually cajoled into putting their trust and their money into the thus-far-futile search for a cure, and have not been told what has been learned about prevention. Without this information, Americans every day unknowingly choose to eat foods that contribute heavily to their risk of cancer.
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> In 1976, the United States Senate Select Committee on Nutrition and Human Needs, under the chairmanship of Senator George McGovern, convened public hearings on the health effects of the modern American diet. After listening to the testimony of the nation's leading cancer experts, McGovern was not particularly delighted with the war on cancer, calling it a 'multi-billion dollar medical failure.'
>
> At one point in the proceedings, McGovern pointedly asked National Cancer Institute director Arthur Upton how many cancers are caused by diet. The head of the largest cancer organization in the world replied 'up to 50 percent.'
>
> McGovern was dumbfounded. 'How can you assert the vital relationship between diet and cancer,' he demanded, 'and then sub-

mit a preliminary budget that only allocates a little more than one percent (of the National Cancer Institute funds) to this problem?'

Dr. Upton responded sheepishly: 'That question is one which I am indeed concerned about myself.'

The problem is that diet is not a 'magic bullet.' It is a way of preventing cancer, but only in rare cases a way of cure. Organizations like the National Cancer Institute are not encouraged to focus much attention on prevention because there is vastly more money to be made in treatment, and far more glamour in the possibility, however remote, of a cure. Attention is further drawn away from prevention by food industries whose products are known to be involved. They apply immense pressure on government and public health organizations to keep them from informing the public as to what is known about dietary prevention. The result is that you and I are continually being told to put our faith and our money into cancer treatment, and into the hope for an eventual cure. We are not told how to keep cancer from happening in the first place.

The tragic result is that we are losing a war we could prevent.[2]

The new food labeling was fought by meat producers because fat consumption guidelines were originally designed to be based on a 2000 calorie diet, of which no more than 30 percent of the calories were derived from fat. Since meat is high in fat, and fat is high in calories, a 2000 calorie diet provides for only modest meat consumption. A compromise was finally reached with the meat industry wherein two diet scenarios will appear in 1994 on food labels: one for a 2000 calorie diet and another for a 2500 calorie diet.[3]

Since there have never been any guidelines set by law, fat percentages have often been computed based on the weight of a serving size. Producers of high fat foods can make some of the fattiest foods appear a delight to dieters by referencing their calculations to weight. But a little thought will show this method to be lean on meaning and fat on deception.

All living things depend on food for their survival. Those who produce high-fat foods would have us believe it is the weight of the food we consume, rather than the energy it provides, which is essential. But anyone with common sense can see energy, not weight, is the

issue—it's **what** is eaten that counts, not **how much**. You choose: four pounds of apples a day for the rest of your life, or one to two pounds a day of varied foods. Take whole milk for example: An 8 ounce serving has a "weight" of 226.8 grams (grams are really a measure of mass not weight). Only 9 grams are fat. So fat as a percentage of weight is:

$9/226.8 \times 100 = 3.97\%$, which is low (4% assuming only one significant figure). To avoid any confusion all answers will be given to two significant figures unless otherwise stated.

A more meaningful calculation, however, is fat as a percentage of calories. An 8 ounce serving of whole milk has 150 calories. One gram of fat has 9 calories. Since there are 9 grams of fat, this gives a total of 9 grams $\times$ 9 calories/gram = 81 calories. The percentage of fat from calories is: $81/150 \times 100 = 54\%$, which is close to fourteen times greater than the previous answer.

Computing the percentage of fat from total calories is exactly the information you and I need to know to make healthy decisions. If 50 percent of our available energy is in the form of fat, the risk of heart disease and cancer is substantially higher. The present government guideline recommends no more than 30 percent of calories from fat, and there are those in the medical profession who advocate numbers as low as 20 and even 10 percent.[4]

Even maintaining a diet with as much as 30 percent fat from calories will require Americans to reduce their consumption of animal foods appreciably. This, as cited above, has caused the meat industry much concern, which is why they lobbied against nutrition labeling. A higher caloric diet, however, allows for a higher level of fat consumption.

As stated previously, the new food labels will have guidelines for two diet scenarios, they will be:

1. A diet based on 2000 calories per day, where no more than 30 percent of the calories are derived from fat.
2. A diet based on 2500 calories per day, where no more than 30

percent of the calories are derived from fat.

The important questions for us to ask are:

If one gram of fat has 9 calories, how many grams of fat can be eaten without exceeding the 30 percent limit, given a 2000 calorie diet and a 2500 calorie diet?

Thirty percent of 2000 calories represents the total number of fat calories permitted each day. This number divided by 9 calories per gram gives the number of grams of fat that can be eaten each day.

Thus: 2000 calories × .30 = 600 calories,
600 calories/9 calories per gram = 66.67 grams

Most labels presently give the number of grams of fat per serving so it is only necessary to add up your daily total and keep it under 67 grams.

A diet based on 2500 calories per day gives:

2500 calories × .30 = 750 calories,
750 calories/9 calories per gram = 83 grams.

The new labels will also give the percentage of fat in a serving, based on 2000 calories per day. For example, if a serving has 13 grams of fat, this represents 20 percent of your total fat intake for the day (13/66.67 × 100 = 20%). Additionally, the label will provide the number of calories in a serving and the number of calories from fat. This will be yet another measure of fat content. The above item may have 260 calories a serving, of which 120 are from fat. This tells us that the item, though providing only 20 percent of our fat intake for the day (120/600 × 100 = 20%), is itself 46 percent fat (120/260 × 100 = 46%). In addition to the total fat, both saturated fat and cholesterol content will be listed on food labels in 1994.

Most saturated fat is found in animal products, with the exception of some tropical oils, but only in animal products do we find cholesterol. In dairy products, fat and cholesterol are usually found together. Thus, many low fat items are also lower in cholesterol. This, however, is not true of meat products where cholesterol is found predominately in muscle. Lean cuts of meat, therefore, do not affect cholesterol content. Most people erroneously assume that only red meat poses a cholesterol problem. Ounce for ounce, the cholesterol content of chicken is comparable to that of red meat. The only way to reduce cholesterol consumption is to eat less animal products.

Atherosclerosis, a warning sign for heart attacks, results when cholesterol occurs in plaques along the inner walls of the coronary arteries, closing off the flow of blood to our heart. Low-density lipoprotein (LDL) is commonly called “bad cholesterol,” because of its role in transporting two-thirds of all cholesterol through our circulatory system. Much of this cholesterol ends up adhering to the inner walls of our blood vessels. (The body also has high-density lipoprotein (HDL) which is supposed to remove the accumulating cholesterol from blood vessels, hence it is referred to as “good cholesterol.”) Since saturated fats are found with cholesterol, most medical authorities advise reducing saturated fats to no more than 10 percent of our daily calories, as well as keeping daily cholesterol consumption to within 300 milligrams.[5] (A milligram (mg) is a thousandth of a gram.) Before leaving Problem #1, try the two exercises below:

1. Louis Rich Turkey Bologna is advertised as 90 percent fat free—10 percent fat. A one ounce serving (28 grams) = 1 slice. Fat content is 3 grams and one serving has a total of 45 calories.
a) Compute the percentage of fat by weight and the percentage of calories from fat. (Recall 1 gram of fat has 9 calories.)

Answers: 10.7 percent by weight and 60 percent from calories

Note—All numbers shown on food labels have been rounded-off. Do not expect computed values based on these numbers to yield consistent results for the total calories printed on the label. When computing the total calories, 4 calories per gram for protein and carbohydrates is used, as is 9 calories for one gram of fat—but these figures are also approximated. For example, Kellogg's Nutri-Grain Almond Raisin Cereal has 140 calories per serving. The label lists 3 grams of protein, 31 grams of carbohydrates, and 2 grams of fat. Since both protein and carbohydrates have the same number of calories per gram, we have: $(3+31) \times 4 = 136$ calories + $2 \times 9 = 18$ calories for a total of 154 calories. This is not necessarily an attempt to distort the facts (especially in this example, since a smaller number of calories per serving would appear to increase the percentage of fat in each serving), but it is an example of mathematical confusion. Most companies provide a toll free number on their labels for consumer questions.

b) Assuming a maximum of 67 grams of fat per day, what percentage of total calories from fat would 4 slices of this bologna provide for the day?

Answer: 18 percent

2. A "brown bag" lunch is packed for your child that contains two bologna and American processed cheese sandwiches, each spread with a tablespoon of butter. The sandwiches have one slice of bologna (Louis Rich Turkey Bologna) and one slice of American cheese. One slice of American cheese has 8.9 grams of fat and 106 total calories. One tablespoon of butter has 10.8 grams of fat and 101 total calories.[6]

a) Compute the percentages of fat calories for cheese and butter.

Answers: cheese — 76 percent, butter — 96 percent

Again, due to rounding-off, the answer for butter did not evaluate to 100%. Actually, butter has a very small amount of carbohydrates .06 grams per tablespoon, so it is not entirely fat, but close enough.[7]

b) Compute the total number of grams of fat eaten for lunch.

Answer: 45.4 grams

c) What percentage of the total fat requirement for the day has been consumed? Solve the problem for both a 2000 calorie diet and 2500 calorie diet.

Answers: 68% and 55%

Calculate the above values if a one ounce bag of potato chips and an 8 ounce glass of whole milk are also consumed for lunch. Do you think this lunch is healthy?

Problem #2
Inflation—or How to Compare Apples to Oranges

An article in the Investor's Business Daily (February 18, 1993) titled "Clinton's New War on Drugs" defended drug price increases and the role drug companies play in supporting our economy and lessening hospital costs. The author maintained that government regulation is one of the reasons for higher priced drugs. The focal point of this argument centered around legislation passed in the early 1960s.

In 1962, the Kefauver Amendments (named after Senator Estes Kefauver) were passed. These amendments gave the Food and Drug Administration a greater voice in the approval process for new drugs. The law stated that companies must "prove not only that a new drug was safe—as was required under previous law—but also effective."[8] According to the article, "In 1963, before the law was fully implemented, it took only 2.5 years and $29 million to bring a new drug to market.

By 1990, the drug approval process took 12 years, on average, and more than $231 million ..."[9] It may well be that the drug companies are getting a bum deal by the Clinton Administration, but using the dollar amounts cited above are misleading. The article makes no reference to the effects of inflation for the dollar amounts cited. Comparing "dollars" from different time periods without taking inflation into account is like the proverbial apples to oranges comparison—it makes no sense. Using the figures given above, one concludes the cost to pharmaceutical companies, due to government regulation, has increased by close to a factor of eight (231/29 = 7.96). But this calculation ignores time and inflation—$231 million over **12 years** and $29 million over **2.5 year**s. The article does not indicate in “what dollars” the quoted amounts are given. Is the $231 million figure referenced to 1990 dollars, or is it a sum of all the money spent (ignoring inflation) twelve years prior to 1990? The value of the dollar changed appreciably during this time period. It turns out that this information ($231 million over a twelve year time period) was taken from “a study by Joseph DiMasi, an economist at the Center for the Study of Drug Development at Tufts University.”[10] In an effort to better understand the newspaper quote, I called Joseph DiMasi at Tufts University and asked him exactly how the $231 million was computed. He told me that his study used the time period 1970-1982 and that the $231 million figure was referenced to **1987 dollars**. He was unable to provide any verification of the 1963 figure, as it was not part of his study. Though I’m sure the journalist was sincere in his efforts, not enough information was provided to the reader in the article. Without careful consideration, two erroneous conclusions can quickly be made:

1. As stated above, the cost to drug companies appears to have risen nearly eightfold from 1963 to 1990.
2. On a yearly basis the cost rose from $11.6 million/year to $19.3 million/year—a price increase of almost twofold. (These figures are obtained by dividing $29 million by 2.5 years, and $231 million by 12 years.)

The central question of Problem #2 is: **How much have costs for drug implementation increased between the early 1960’s and the period 1970-1982 in constant 1987 dollars?**

To answer this question we must have an understanding of inflation. Inflation is insidious. Every money civilization has suffered its effects. Recall the problems in ancient Athens discussed in

Chapter Three. Solon’s devaluation of Athenian currency was a risky solution to a perilous problem. Remember, when money is devalued its purchasing power declines; this is no different than inflation.

Inflation in America has averaged about 4 percent over the last ten years (1983-1993). That is, if the value of the dollar is referenced to 1983, it has, on average, lost 4 percent of its buying power each year thereafter. For example, if the inflation rate averaged exactly 4 percent each year, we would have: $\$1.00 \times .04 = 4¢$; this means a 1984 dollar has the purchasing power of 96¢ with respect to a 1983 dollar. Another 4 percent inflation for 1985 would mean 4 percent off of the “96¢ dollar” of 1984. Since inflation is typically calculated from one year to the next, an average inflation rate of 4 percent over ten years would not devalue a dollar by 40¢; the amount is closer to 31¢. To make this point clear, below is the changing value of a dollar over a ten year period assuming constant 4 percent inflation. (All calculations are rounded to the nearest penny.)

Table 6.1

Decline in purchasing power of a dollar at constant 4 percent inflation

Year	**Value of dollar referenced to year 1**
1	100¢
2	96¢
3	92¢
4	88¢
5	85¢
6	82¢
7	78¢
8	75¢
9	72¢
10	69¢

Table 6.1 shows that after ten years at constant 4 percent inflation a “tenth year dollar” will only buy 69¢ worth of goods compared to a

"first year dollar." There are many ways to describe this situation:

1. The buying power of the "tenth year dollar" is at 69 percent (69⁄100) of the "first year dollar."
2. The buying power of a dollar has fallen 31 percent (100-69)/100 = 31⁄100, over a ten year period.
3. If Table 6.1 is assumed to represent inflation for consumer prices, then you would need $1.44 "tenth year dollars" to have the same purchasing power as one "first year dollar" ($1.00/.69 = $1.44).

An appliance in "year one" costs $1000 and the same appliance costs $1500 dollars in "year ten." It appears the price has increased 50 percent. But 1500 "tenth year dollars" are only 1035 "first year dollars" ($1500 × .69 = $1035). The increase in constant dollars is $35 and therefore: 35⁄1000 = .035 or 3.5 percent.

Table 6.2 gives consumer and producer prices in constant 1967 dollars. Since the costs for "producing" new drugs were cited in the newspaper article, this is where our attention will be focused.

Table 6.2

Purchasing power of dollar for producer and consumer prices for 1959-1990 (1967 = $1.00)

Year	Producer Prices	Consumer Prices
1959	1.08	1.15
1960	1.07	1.13
1961	1.07	1.12
1962	1.06	1.11
1963	1.07	1.09
1964	1.06	1.08
1965	1.04	1.06
1966	1.01	1.03
1967	1.00	1.00
1968	0.97	0.96
1969	0.94	0.91
1970	0.91	0.86
1971	0.88	0.82
1972	0.85	0.80
1973	0.78	0.75
1974	0.68	0.68
1975	0.61	0.62
1976	0.59	0.59
1977	0.55	0.55
1978	0.51	0.51
1979	0.46	0.46
1980	0.40	0.41
1981	0.37	0.37
1982	0.36	0.35
1983	0.35	0.34
1984	0.34	0.32
1985	0.34	0.31
1986	0.34	0.30
1987	0.34	0.29
1988	0.33	0.28
1989	0.31	0.27
1990	0.30	0.26

Source: Facts and Figures on Government Finance 1992 Ed.

Based on 1967 dollars, a "dollar" in 1963 had $1.07 worth of purchasing power. Similarly, a 1987 dollar had $.34 worth of purchasing power (see Table 6.2). To answer the original question, one must compute the value retained by a dollar between 1963 and 1987. (We can't be sure to what year the $29 million figure was referenced. But since the value of the dollar was fairly constant during the early 1960s, according to Table 6.2, using the value of the dollar in 1963 is reasonable.) We proceed as follows: $.34/$1.07 = .32 or 32 percent. This means every 1987 dollar is worth only 32¢ compared to each 1963 dollar. Therefore, in 1963 dollars, (231×10^6) 1987 dollars have a value of $.32 \times \$231 \times 10^6 = \$74 \times 10^6$.

So how much has the cost risen to bring a drug to market today as opposed to the early 1960s? There are several ways to answer this question. The increased cost is $74 million - $29 million or $45 million. Or, the cost has increased by a factor of 2.55 since $74 million/$29 million = 2.55. These calculations, however, do not tell the whole story. The element of time must also be considered. In the first case we are talking about 29 million 1963 dollars over 2.5 years, and in the latter case, 74 million 1963 dollars over 12 years. The best way to compare "apples to apples" is to calculate the average cost per year. Twenty-nine million dollars over a 2.5 year period works out to $11.6 million/year ($29 million/2.5 = $11.6 million/year). Likewise, $74 million over 12 years works out to $6.2 million/year ($74 million/12 = $6.2 million/year). This shows that the yearly cost to producers in constant dollars is about half of what they were paying in 1963! This is arrived at by the computation: $6.2 million/11.6 million × 100 = 53%. The actual cost has fallen by 100% - 53% = 47% on a yearly basis.

Alternatively, the problem can be done from the point of view of constant 1987 dollars. We proceed as follows: If 32¢ in 1963 buys the same amount as a dollar in 1987 then it takes 3.13 1987 dollars to equal one 1963 dollar (1.00/.32 = 3.13). Therefore, $29 million in 1963 is the same as $90.8 million in 1987 ($29 million × 3.13 = $90.8 million). The cost per year in 1987 dollars is:

$90.8/2.5 = $36.3 million/year for 1963 and
$231/12 = 19.3 million/year for 1987

And just as before the cost per year to drug producers is 53 percent of what it was in the early 1960's (19.3 million/36.3 million × 100 = 53%).

So which is it? Have drug costs increased or decreased? The answer depends upon how you look at the problem! Which tells us that a little skepticism about the conclusions people reach using numerical information is healthy. If inflation isn't mentioned and dollar figures are quoted for different years, be on your guard. Remember the vital role time can play in interpreting one situation in more than one way. Before leaving "Problem #2", try doing the exercises below:

1. A car, comparable to one that cost $6000 in 1980, sells for $12,000 in 1993. (Use Table 6.2 for Consumer Prices, assume the purchasing power of the dollar for 1993 is 23¢.)
a) What is the cost in constant 1980 dollars?
b) Using constant dollars, determine the percentage increase in price.

Answers: a) $6732 b) 12%

2. Computer power that cost $2000 in 1986 sells for $500 in 1993.
a) What is the cost in constant 1986 dollars?
b) How much has the price gone down in 1986 dollars?
c) By what percentage has the price of computing power gone down from 1986 to 1992?

Answers: a) $383 b) $1617 c) 81%

Note—Using 3 significant figures $1617 should be $1620

Problem #3
A Taxing Situation

President Clinton has proposed an energy tax (as of early 1993) to help offset the federal deficit. The tax would levy an extra 59.9¢ per million BTUs (British Thermal Units) on gasoline and heating oil, and 25.7¢ per million BTUs on natural gas, coal, and nuclear energy.

There is always a flurry in Congress and among the general public over any form of tax increase; understandably, since "tax and spend" is as dangerous as "cut and spend." Regardless, it seems inevitable that the American people will be asked to help pay down the deficit by increased taxes in one form or another. The natural question for most of us is: How much? Though we can never be certain what the final cost will be, the problem below may provide for a good start.

Based on the previous figures, what should the average American expect the President's energy tax to cost if it is implemented?

There is a "fast and dirty" way to get an upper limit on this amount by finding the BTU consumption per capita for the United States. This information can be found in an Almanac or can be easily calculated if not given directly. Most Almanacs give the total number of BTUs used per year in the United States. If the book does not explicitly give the consumption per capita, it can be found by dividing the total consumption in BTUs by the population of the United States. This answer would be very rough since it does not separate out energy use by commerce, industry, and the military. It does, however, bring up an interesting point.

Usually, this kind of tax is based on primary energy expenditures that the average person must pay, such as gasoline for automobiles and home energy fuel. However, since every product that is sold depends on energy for its creation and relies on energy to bring it to market, producers will no doubt pass on their increased costs to consumers. This part of the tax is hidden from view. Indeed, calculating the tax burden based on per capita energy usage may represent a rough estimate for the **maximum** outlay each American can expect to pay. We will do this, as well as a computation for primary energy expenditures based on gasoline and home fuel use. Computing the cost for the latter, however, requires a bit more effort, since some homes rely on oil and others natural gas.

Before beginning the mathematical part of the work, it would be beneficial to digress for the moment and discuss the concept of energy, since many readers may not understand what a BTU measures.

Measuring Energy

Energy is an abstract concept. It is associated with all things in nature and manifests itself in a myriad of ways. Scientists can measure and use energy, but no one really understands what it is.

It wasn't until well into the nineteenth century that scientists understood that heat is a form of energy. The Englishman James Prescott Joule (1818-1889) showed that mechanical work and heat are equivalent. He did this by attaching a weight on a string that was strung over a pulley and connected to a paddle wheel. This allowed the weight to turn the paddle wheel when released. Before allowing the weight to fall he measured the temperature of the water. After the weight had fallen he measured the temperature again and found an increase due to the frictional effects of the paddle wheel in the water. Knowing the temperature increase and the amount of water in the bucket permitted him to formulate a relationship between mechanical work and heat. Since the temperature increase of the water due to the rotating paddle was indistinguishable from heating the water with a flame, the two processes (mechanical work and heat) were seen as equivalent.

Had scientists initially understood the relationship between work, heat, and energy, there may never have been a reason for calories or BTUs, since both work and heat are a measure of energy. Nor is it only correct to express the energy content of food in calories, it is more a matter of history; food could just as well be represented in BTUs. Historically, both the calorie and BTU were defined by the amount of heat necessary to raise a given amount of water to a fixed temperature. In the case of the calorie, it was defined as the amount of heat needed to raise one gram of water (about a twenty-eighth of an ounce) from 14.5° to 15.5° Celsius. The more common food calorie is called a "big calorie" and is defined as one thousand of the calories described above (1 **kilo**calorie = 1 food calorie). A BTU is similarly defined as the heat necessary to raise one pound of water from 63° to 64° Fahrenheit. The calorie has been used more in laboratory work, whereas the BTU is employed for industrial and engineering needs.

In 1948, scientists honored James Presscott Joule by using his name to represent the standard energy unit in the International System of Units. Presently, the calorie and BTU are being phased out because their definitions are now viewed as archaic. Work, heat, and energy are increasingly being expressed in Joules today.

The Joule is not a fundamental unit of measurement. It owes its definition to the product of force and distance. One Joule is the amount of energy required when a force of one Newton acts over a distance of one meter. A Newton is a unit of force or weight in the metric system; one pound equals 4.45 Newtons and a meter is about 39 inches. Therefore, lifting a weight of slightly less than a quarter-pound a distance of 39 inches off the ground requires one Joule of energy. The energy released in burning one wooden match is about one BTU; this in turn is equal to 1,055 Joules (written as 1055 J). Clearly, a Joule is a small unit of measurement. One small calorie equals 4.186 J, and one food calorie equals to 4,186 J. Given the above information, it requires only one step to show that 1 food calorie is equal to about 4 BTUs: 4,186 J/cal/1,055 J/BTU = 3.97 BTUs/cal. In other words, a slice of bread with 80 calories can be expressed as approximately 318 BTUs ($80 \times 3.97 = 317.6$) or 334,880 J ($4186 \times 80 = 334880$ J).

By using one standard reference, the Joule, we see more clearly that energy merely takes different forms while performing various tasks. The same energy units that measure the dynamics of a locomotive can be used to analyze the metabolic rate of a living being.

Calculating the Tax on Gasoline and Home Energy Fuel

Returning to the original problem, we first compute the added cost of operating an automobile. *The Universal Almanac* tells us that, "An engine burning 8 gallons of gasoline releases 1 million (10^6) BTU."[11] Since the proposed tax on gasoline is 59.9¢ per 10^6 BTUs, it

means an increase of 59.9¢ for each 8 gallons of gasoline purchased. If a car is driven an average of 12,000 miles a year and the average gas mileage is 20 miles per gallon, then 600 gallons must be purchased each year ($(12 \times 10^3)/(2 \times 10^1) = 600$). Each 8 gallons of the 600 represents another 59.9¢. Since there are seventy-five 59.9 units in 600 gallons ($600/8 = 75$), the tax per year amounts to 75×59.9¢ = \$44.93 = \$45. (Recall we are using two significant figures.)

A more revealing number would be the actual increase in cost per gallon of gasoline. Since the increased cost on each gallon is not dependent on the number of miles driven nor the gas mileage received; it therefore gives a more universal expression of the tax increase, one that is more easily understood than the tax on BTUs. The figures above permit two very quick ways of calculating the added tax per gallon.

Very simply, since 8 gallons correspond to one million BTUs, dividing 59.9¢ by 8 gallons gives 7.5¢ per gallon extra. Alternatively, we can divide the increased cost per year (\$44.93) by the number of gallons purchased in a year (600). This also yields 7.5¢ per gallon to two significant figures. So if you're presently paying \$1.00 per gallon for gasoline, expect to be paying about \$1.08 if this tax is enacted.

Next, a calculation for home fuel costs must be done with respect to both natural gas and heating oil. According to The Universal Almanac, the average BTU consumption per household in 1987 was 100 million BTUs.[12] This includes all energy use—space heating, appliances, air conditioning, and water heating. Three calculations: only gas consumption, only oil consumption, and an average of both, will give reasonable figures for the added tax burden on maintaining a home.

Recall that natural gas is to be taxed at 25.7¢ /10^6 BTUs, and that heating oil is to be be taxed the same as gasoline at 59.9¢/10^6 BTUs.

Case 1: If a household relies only on natural gas the added tax will be:
25.7¢/10^6 BTUs $\times 100 \times 10^6$ BTUs = \$25.70.

Case 2: If a household relies only on home heating oil the added tax will be: $59.9¢/10^6$ BTUs $\times 100 \times 10^6$ BTUs = \$59.90.
Case 3: If both natural gas and home heating oil are used equally the added tax for the average will be: (\$25.7 + \$59.9)/2 = \$42.80.

The total cost to the consumer is the added cost of gasoline needed to fuel at least one automobile and the added cost of home heating fuel. (Recall that \$44.93 was the additional cost for maintaining one automobile.) The total costs are given for each of the three cases below:

1. \$44.93 + \$25.70 = \$70.63 (\$71 to the nearest dollar)
2. \$44.93 + \$59.90 = \$104.83 (\$100 assuming two significant figures)
3. \$44.93 + \$42.80 = \$87.73 (\$88 to the nearest dollar)

Finding the Per Capita Tax

The BTU tax is all inclusive. Farmers and manufacturers alike depend on energy in one form or another. If their additional costs are passed on to consumers then each of us will pay a share of the tax based on the total energy consumption for the United States. This is a worst case scenario for the consumer.

In 1988, the U.S. consumed 80,000 trillion BTUs (80 quadrillion BTUs or 80×10^{15}).[13] Using a population of 250×10^6, the per capita BTU consumption is: (80×10^{15} BTUs)/(25×10^6) = 320×10^6 BTUs/person. Approximately 42 percent of the energy used in the U.S. in 1988 was petroleum based.[14] Therefore: 42% of the 320×10^6 BTUs will be evaluated at 59.9¢ and the remaining 58% will be evaluated at 25.7¢ .

Calculation for Petroleum use at 42 percent:

$.42 \times 320 \times 10^6$ BTUs = 134.4×10^6 BTUs,
134.4×10^6 BTUs $\times$ 59.9¢/10^6 BTUs = \$80.5

Calculation for all other energy use at 58 percent:

$.58 \times 320 \times 10^6$ BTUs = 185.6×10^6 BTUs,
185.6×10^6 BTUs $\times$ 25.7¢ /10^6 BTUs = \$47.7

Total cost for both forms of energy:

\$80.5 + \$47.7 = \$128.2 = \$130 to two significant figures.

Therefore, the highest average tax for each consumer should not exceed \$130 in the worst case. Of course, if you live in a large house, drive in excess of 12,000 miles a year, and make many and large purchases, your "worst case tax scenario" will be greater.

There is a large difference between per capita BTU use and per capita end-use. The difference between the two amounts is a measure of the waste inherent in the system. No process in nature is 100 percent efficient. It turned out that only 244×10^6 BTUs/person (end-use) of the 320×10^6 BTUs/person produced in 1988 was actually usable. The rest was lost in the generation, transmission, and distribution processes, where energy losses result from the creation of unwanted heat.

Fortunately for us, our utility bill reflects only our usage, we are not charged for transmission or distribution losses. However, we do pay for the energy losses (in the form of heat) from our appliances and autos. There is a lesson to be learned here. The more efficient our "end-use" of energy, the less of it is consumed and more money is saved. This makes fuel efficiency and conservation major players in saving money.

Looking a Little Deeper

Figure 6.1 shows car fuel efficiency standards for each year during the interval 1978-1991.[15] A simple average for this thirteen year period is 24.5 miles per gallon. But the average should really be "weighted" since the number of cars is not evenly distributed each year. This fact, in addition to the toll that wear and tear have on gas mileage, and the fact that fuel efficiency is measured under optimum conditions, is the reason 20 miles per gallon was chosen. See Figure 6.1 below. It should also be noted that Figure 6.1 pertains only to U.S. made automobiles. The contribution from imported cars slightly increases fuel efficiency each year. The weighted average U.S. fuel efficiency for passenger cars (assuming 55% city driving and 45% highway driving) as of 1988 was 19.17 miles per gallon.[16] If we factor in the slight increase in mileage due to imported vehicles, 20 miles per gallon is a reasonable estimate.

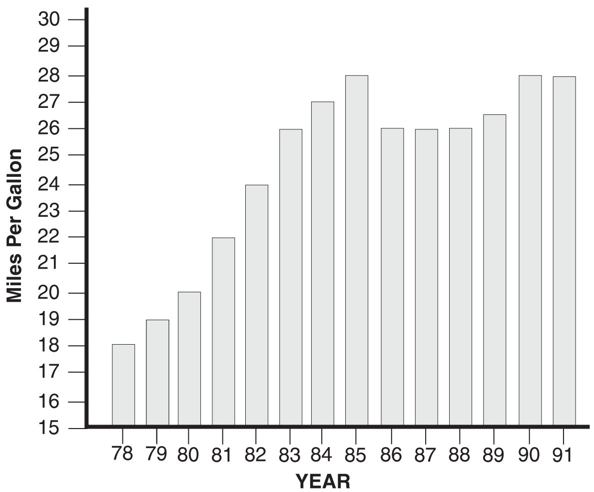

Figure 6.1
Graph showing U.S. car fuel standards

Source: Adapted from, *Environmental Almanic*, compiled by World Resources Institute.

Had higher fuel efficiency standards been set during previous years, how much money could have been saved in 1991? Let's assume that higher standards would have boosted the present 20 miles per gallon to 24 miles per gallon and calculate the savings. In 1990, America had 145 million passenger cars. The average increase from 1985 to 1990 was approximately two million cars per year. Using this

to extrapolate to 1991, gives an estimated 147 million automobiles.[17]

Since 24 miles per gallon reduces gas consumption to $\frac{5}{6}$ of what it is at 20 miles per gallon ($\frac{20}{24} = \frac{5}{6}$), each car consumes only 500 gallons per year ($\frac{5}{6} \times 600 = 500$). (Recall that at 20 miles per gallon fuel consumption for an automobile driven 12,000 miles per year is 600 gallons.) If each car uses 100 gallons less fuel every year, at an average cost of $1.00 per gallon, this is a savings of $100.00 per auto. This works out to 147×10^6 cars $\times$ $100/car = $$14.7 \times 10^9$, or $14,700,000,000.

Furthermore, if we perform a similar calculation (with the same assumptions) based on the per capita use for **all transportation**, the final figure would come to about $28 billion in savings. This number was calculated as follows: 42% of the fuel used in America is in the form of petroleum and about 62% of this is for transportation of all kinds.[18] If the per capita fuel consumption is 320×10^6 BTUs/person, then the amount of energy used by each American for transportation is:

320×10^6 BTUs/person $\times .42 \times .62 = 83.3 \times 10^6$ BTUs/person. Since 10^6 BTUs are equivalent to 8 gallons of gasoline, then $83.3 \times 8 = 666.4$ gallons are used for transportation, on average, for each person per year. If only $\frac{5}{6}$ the amount of gasoline is used, then each person consumes only 555.3 gallons of gas ($666.4 \times \frac{5}{6} = 555.3$ gal), thus saving on the cost of 111.1 gallons. The present population is 2.5×10^8, so at $1.00 per gallon, the savings equals $$111.1 \times 2.5 \times 10^8 = \$277.8 \times 10^8$, = $$27.8 \times 10^9$ or about $28 billion. One can think of this number as a rough estimate of what the maximum savings for the American public in 1991 could have been with a modest gain in fuel efficiency over the years. Such an increase in fuel efficiency is by no means unreasonable today. "Indeed, one study covered in the UCS (Union of Concerned Scientists) book, *Steering a New Course*, concluded that fuel economy could cost-effectively be increased to over 40 mpg without sacrificing size or acceleration."[19] Additionally, the UCS and other environmental organizations believe "...there is tremendous opportunity for further efficiency gains without any sacrifice in safety."[20] Auto manufacturers usually cite decreased consumer safety as an argument against greater fuel efficiency standards.

Though Americans may have to endure higher energy taxes in order to bring down the deficit, there is no reason creative steps cannot be taken to alleviate some of this burden. Increasing fuel efficiency standards may dramatically help (over a period of years) balance out the cost of additional energy taxes. In the end, Americans will have more money in their pockets, which makes for a healthier economy. Before leaving Problem #3 do the exercises below:

1. Estimate the revenue the government can expect to raise from private energy use, assume two cars per household and one hundred million households.

Answer: Over $13 billion

2.a) Estimate the revenue again, but this time use the per capita information based on 320×10^6 BTUs/person, assuming equal shares of petroleum and all other energy sources.
b) Recalculate part (a) using petroleum as the only energy source.

Answers: a) $32 billion b) $50 billion

The above two problems make it clear that figures read in newspapers and quoted by analysts can easily be misunderstood unless the initial conditions are emphasized. How would these numbers change if the above calculations were done based on the end-use per capita figure?

3. It is interesting to note that the U.S. produced only 67 quadrillion BTUs in 1988;[21] find the percentage of energy the U.S. imported for this year.

Answer 16%

There is much more that can be said about all three of the problems we have discussed. Each one has a complex mixture of social, political, and mathematical components. When problems are placed within the perspective of history, culture, and science, we can see more clearly that there are few truly simple problems. Only by understanding all of the different attributes of a problem can lasting and positive solutions be found.

The nineties hold great promise for personal health, efficient resource use, and improving our economy. But we must understand those factors that influence our personal and political choices, and appreciate the insight science and mathematics can contribute to our decisions.