CHAPTER ONE

The Problem

No subject of human concern can exist in isolation without becoming a hopeless wasteland of rules and facts. If history is a medium for charting humanity's unfolding; if literature is driven by existential themes and the social, economic, and cultural bias of the times; if art and music are expressions of the landscape of human development; then mathematics must also be seen, interpreted, and taught with the same passion and wholeness that makes any other human activity clear and alive. Colin Wilson, who has authored many intriguing books on the human condition summed it up well when he wrote, "Boredom cripples the will. Meaning stimulates it."[1]

So much of what is given to students in mathematics is without context, history, or purpose. Much of mathematics education is analogous to memorizing thousands of foreign words with the promise of someday forming meaningful sentences. Looking honestly at how the subject is presented, it is little wonder that few students see its relevance. We have taught the subject in isolation, paying lip service to meaning by creating a context that is strained and unconvincing at best.

Students are expected to operate symbolically with algebra without a clear understanding of decimals and fractions. Even those individuals who can perform basic operations do so because they have remembered a series of steps (an algorithm) for solving the problem. The question, "Why does this algorithm work?," is rarely considered. Most people have spent so much energy keeping all the rules straight in their minds, that the rationale behind the rules is overlooked. Nor is this only symptomatic of mathematics. I have known students to complete a college physics course with an A who knew no more than how to plug numbers into formulas. Others who managed B's and C's were still remarkably medieval in their view of physical reality. It is just as common for students without an understanding of basic algebra to successfully complete their requirements in calculus. They learned long ago that memorizing procedures and jumping through the appropriate "hoops" would yield passing grades. This is not to say that memorizing certain information and knowing some rules are contrary to good education. But the prevalent mechanical and rule-based approach throughout most of elementary and secondary school is devoid of comprehension and wonder, and instead, filled with fear and anxiety.

This reminds me of the science fiction classic, "*The Day the Earth Stood Still.*" In the movie, Michael Rennie (the benign, though powerful alien) gives Patricia Neal (the above average human) three alien words to memorize while they are being pursued. Rennie makes it clear to Neal that if anything happens to him his bodyguard robot will wipe out our violent, primitive species. He's killed moments later, and mankind will be safe only if Neal is successful in reaching the robot and uttering the three words. Now I've seen this film several times (it is, after all, a classic) and for the life of me, and the planet, I can never remember the three alien words. Each time my memory fails, I am struck by the same thought. I picture myself in front of Rennie's robot mumbling incoherently, trying desperately to regurgitate the proper sequence of sounds. The robot (and this is the really pathetic part) amused by my stupidity, bypasses me, and goes on to make short work of the rest of the planet. My only consolation is being spared the embarrassment of having anyone know the fatal role I played.

So much of the rule-based approach and lack of context in mathematics is paralleled in this story. There is the great fear of incorrectly solving the problem and looking foolish. Students do not want anyone to know how "stupid" they are. They feel totally lost and assume everyone knows more than they do. When they are shown how to solve the problem they often admit, "I don't understand what's happening." This is a reasonable response since they are given only instructions, not explanations, which leads to feelings of fear and inadequacy. A similar situation (though the predominate feeling here is boredom) occurs in history classes when the subject is debased into dates and names. Few students can see the currency in studying, "markers in time and lots of dead people." The person who conveyed this to me also insisted that studying what happened in the past had nothing to do with the present! Unfortunately, poor student instruction is the first step in a chain reaction that ends with ill-trained teachers. The entire process forms a closed repetitive loop, much like "the chicken or the egg" scenario.

In preparation to teach secondary school I took several education courses to gain certification. In one of these courses, "Reading in the Content Area," the teacher read us a story about an elderly woman who loved her cats so much that when they died, not being able to part with them, she put them into a huge freezer. The professor then began explaining various ways to incorporate the story into our curriculum. When my turn came, he suggested I have my students count the number of dead cats. I was teaching gifted algebra at the time! What a discouraging example of the kind of advice being offered to would-be mathematics teachers.

Given the state of mathematics instruction, it is not surprising that large numbers of people see no point to the subject. There are, however, those rare individuals who find the subject to be immensely interesting. Many others could also, but it must be realized that a love of learning and a desire to see deeper cannot exist unless a person is charmed into the effort.

This route is best taken from an inductive approach: beginning with the specific (concrete) and working toward the general (abstract). But if the concrete is stripped of context, and reduced to only the rules-of-the-game, most people will forever be amazed by anyone's scholastic interests.

Sadly, it is often too easy to condition people to see only the most superficial aspects of mathematics. And once such a mind-set is created, it's hard to undo. It's like convincing someone that all there is to an iceberg is the part seen above water. The idea to look deeper or give credence to the fantastic possibility that nine-tenths of the iceberg is below the surface would seem too incredible. What's worse, those few who would espouse such a concept would be chastised and ridiculed.

Several years ago I was asked to address a group of middle school teachers. The subject of the talk revolved around computer and calculator use in the classroom. I was an advocate for the use of both of these tools at the earliest levels of education. A number of teachers felt this to be dangerous since in their minds I was abandoning learning and replacing it with key-stroke fluency. In response to this, I asked, "How many of you teach or have taught students division with fractions?" Everyone raised their hands. I went on and asked, "How many of you feel that your students would be short changed if you were to use calculators solely in teaching this manipulation?" (Texas Instruments had just come out with a calculator that handled fraction arithmetic.) Most thought it a bad idea—many because they felt the children would not really be learning the process, and a small percentage voiced concerns that their roles as teachers were being reduced. I then asked, "Who can tell me why when dividing by fractions we are instructed to invert the denominator and then multiply it by the numerator?" In other words, why does this work? No one answered me.

For some, I think the question had never crossed their minds. Their students were expected to comprehend a series of mechanical steps without reassurance that there was a logical reason beneath the superficial rule. At worst, if the calculator was used as poorly as the handwritten mechanical approach, all the students would be doing is substituting one set of rules for another.

Instead of instructing the students to invert the denominator and then multiply this inverted number by the numerator (for example, $\frac{3}{5} \div \frac{2}{7} = \frac{3}{5} \times \frac{7}{2}$) we would be saying: enter the top fraction, then press the divide symbol, then enter the bottom fraction, now press the equal sign. The advantage to the second set of mindless instructions is that the magic of turning the denominator upside down is done away with. Since students are not given an explanation for inverting, it only serves to reinforce their notion that mathematics is truly mysterious.

There is, however, an added advantage to using the calculator. The instructor could introduce division with fractions by having the students do several multiplication and division problems with fractions on their calculators. (See Chapter Five for an explanation of operations with fractions.) Say for example: $\frac{2}{3} \div \frac{7}{5}$, followed by $\frac{2}{3} \times \frac{5}{7}$, and $\frac{3}{4} \div \frac{2}{11}$, followed by $\frac{3}{4} \times \frac{11}{2}$. Consider these four problems as two sets, each set containing first a division problem followed by a multiplication problem. The students would record their results and be asked if they noticed any pattern. Most would quickly see that the two problems within each set have the same answer. The students would then have an opportunity to investigate the patterns in each set of problems. Many would realize that one can turn a division problem into a multiplication problem by multiplying the denominator of the bottom fraction by the numerator of the top fraction and vice versa.

The instructional advantage is in allowing students to discover the rule for themselves and setting the stage for the more meaningful question—*Why does this work*? This process can, of course, be achieved without the use of a calculator. The calculator, however, focuses attention on the evaluation process, since less time is spent with the mundane task of manipulating the numbers. This is advantageous because information that is given too slowly loses its meaning as surely as information given too rapidly is never received. We lose comprehension if information is not received within a narrow spectrum of time and space. If you try reading the next paragraph at a rate of one word every 10 seconds, the increased time will inhibit understanding; the individual words will become more pronounced than the content of the paragraph.

Similarly, when information is too spread out in space, understanding can also be compromised. There is the interesting story of pilots who noticed large drawings carved into the Peruvian landscape. The drawings are believed to be quite ancient and have stirred all manner of hypotheses, such as alien landing strips. Such imagination stems from the fact that though the markings are indiscernible at ground level, they are clearly obvious from the air. Hence, the drawings have meaning only from a context of height.

The point is to recognize the difference between information acquisition and the information itself. The manner in which ideas are structured and presented helps determine whether people will listen and learn. Unfortunately, too many people hear nothing today.